

Some mechanisms of “spontaneous” polarization of superfluid He-4

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Previously, a quantum “tidal” mechanism of polarization of the atoms of He-II was proposed, according to which, as a result of interatomic interaction, each atom of He-II acquires small fluctuating dipole and multipole moments, oriented chaotically on the average. In this work, we show that, in the presence of a temperature or density gradient in He-II, the originally chaotically oriented tidal dipole moments of the atoms become partially ordered, which results in volume polarization of He-II. It is found that the gravitational field of the Earth induces electric induction $\Delta\varphi \sim 10^{-7} V$ in He-II (for vessel dimensions of the order of 10 cm). We study also the connection of polarization and acceleration, and discuss a possible nature of the electric signal $\Delta\varphi \approx k_B\Delta T/2e$ observed by A. S. Rybalko in experiments with second sound.

Key words: helium-4; electrical activity; dipole moment; acceleration.

1 INTRODUCTION

In a series of fine experiments, A.S. Rybalko, E.Ya. Rudavskii, S.P. Rubets, *et al.* obtained a number of interesting results testifying that the atoms of superfluid He^4 possess electric properties [1]–[4]. In studies of both a standing second-sound half-wave in He-II [1] and torsional oscillations of a film of He-II [2], the alternating electric voltage U synchronous, respectively, to the second sound and torsional oscillations was observed. This voltage was not related to external electromagnetic fields (in experiments [1], the external voltage was present and is supplied to a heater, but its frequency is twice less than that of the observed signal U) and can be a consequence of the volume polarization of He-II. The effects [1, 2] were not explained up to now, though the attempts to elucidate the experiment with second sound [1] were made in a number of works [5]–[9].

Free atoms of He^4 create no electric field far from themselves, because they have zero charge and zero dipole and multipole moments. However, an atom of helium, being surrounded by other atoms, can acquire a dipole moment (DM), which follows from the tidal

mechanism [10]–[13, 8], by which a DM is induced by the interaction with neighboring atoms (we call the mechanism “tidal”, since the deformation of electron shells of atoms in this case reminds gravitational tides). In what follows, we will show that a dielectric is polarized due to the gradient of concentration n or temperature T . The effect arises at the consideration of the interaction of atoms. Also we will study the connection of polarization and acceleration. The idea of inducing the polarization by the concentration gradient was earlier considered in [12, 6, 14, 15]. Below, we will carry on a more exact analysis and determine the volume polarization of He-II in a second-sound wave.

2 POLARIZATION OF He-II DUE TO A GRAVITATIONAL FIELD AND THE GRADIENTS OF DENSITY AND TEMPERATURE

It is obvious that a single atom freely falling in a gravitational field \mathbf{g} is not polarized, since the gravity force causes the same acceleration \mathbf{g} of the nucleus and electrons of the atom.

Consider a dielectric (He-II), being at rest in a gravitational field. The gravity force acting on every atom of a dielectric in the equilibrium state, should be balanced by the difference of interatomic forces which act on the given atom from the side of neighboring atoms. This means that, in this case, the concentration gradient must exist in the dielectric. We now evaluate the polarization of He-II induced by the gravitational field. We start from the equations of two-fluid hydrodynamics [16, 17]

$$D\mathbf{v}_s/Dt = -\nabla\mu + \mathbf{g}, \quad (1)$$

$$\rho D\mathbf{v}/Dt = -\nabla p + \rho\mathbf{g}, \quad (2)$$

$$dp = \rho d\mu + SdT. \quad (3)$$

In the absence of macroscopic motions ($\mathbf{v}_n = \mathbf{v}_s = 0$), we get

$$\nabla\mu = \mathbf{g}, \quad \nabla p = \rho\mathbf{g}. \quad (4)$$

Let us consider that $\rho = \rho(p, T)$, and $\nabla T = 0$ in the stationary case. Then

$$\nabla p = \frac{\partial p}{\partial \rho}|_T \nabla \rho \approx \frac{\partial p}{\partial \rho}|_S \nabla \rho = u_1^2 \nabla \rho. \quad (5)$$

Relations (4) and (5) allow us to determine $\nabla \rho$ which compensate the gravity force:

$$\nabla \rho = \rho\mathbf{g}/u_1^2. \quad (6)$$

We direct the Z axis upward, so that $\mathbf{g} = -g\mathbf{i}_z$. Relation (6) implies that the mean distance between atoms of He-II, \bar{R} , depends on the height as

$$\frac{\partial \bar{R}}{\partial z} = \frac{\bar{R}g}{3u_1^2}. \quad (7)$$

Let us position the coordinate origin at the middle of the layer of He-II. Then relation (6) yields the density distribution for helium

$$\rho = \rho_0 e^{-gz/u_1^2}. \quad (8)$$

At $z = 0$, ρ is equal to the mean density ρ_0 (0.1452 g/cm^3 at $T \lesssim 1.3 \text{ K}$ and the saturated vapor pressure). At the height characteristic of experiments, $z = 10 \text{ cm}$, we have $\rho = \rho_0(1 - 1.6 \cdot 10^{-5})$, i.e. the difference between ρ and ρ_0 for the real size of a vessel is insignificant.

We now evaluate the polarization of He-II. It was shown quantum-mechanically in works [10, 11, 13] that each of two quiescent interacting atoms of He^4 , which are located at a distance R from each other, acquires a tidal DM (TDM) which is directed by the minus to another atom and equal to

$$\mathbf{d}_{tid} = -D_7 |e| \frac{a_B^8}{R^7} \mathbf{n}, \quad (9)$$

where $\mathbf{n} = \mathbf{R}/R$ is the unit vector directed to another atom, and $a_B = \frac{\hbar^2}{me^2} = 0.529 \text{ \AA}$ is the Bohr radius. The quantity $D_7 \approx 18.4$ according to [10, 11], whereas simpler calculations in [13] give $D_7 \approx 25.2 \pm 2$. Taking into account both of results, we accept

$$D_7 \simeq 23 \pm 5. \quad (10)$$

It is convenient to represent \mathbf{d}_{tid} in the form

$$\mathbf{d}_{tid} = -d_0 \frac{\bar{R}_0^7}{R^7} \mathbf{n}, \quad d_0 = D_7 |e| \frac{a_B^8}{\bar{R}_0^7} \simeq 1.88 \cdot 10^{-5} |e| \text{ \AA}. \quad (11)$$

In liquid helium, each atom is surrounded by many other atoms; therefore, its electronic cloud is subject to many deformations, and DM is equal to the sum of DMs induced by all its neighbors. In this case, the total tidal DM of the atoms is equal to zero in view of their irregular location. However, if a density or temperature gradient is present in He II, TDMs of the atoms become partially ordered leading to an overall polarization in He II (see below).

Due to the gradient of ρ along z , the mean distance \bar{R}_1 from the given atom of He-II to the adjacent lower one is somewhat less than the distance \bar{R}_2 to the adjacent upper atom. According to (7) and (11), each atom of helium acquires the noncompensated mean (over the time or atoms) DM in this case:

$$\mathbf{d}_g = \mathbf{i}_z d_0 \left(\frac{\bar{R}_0^7}{\bar{R}_1^7} - \frac{\bar{R}_0^7}{\bar{R}_2^7} \right) \approx -\frac{7d_0 \bar{R} \mathbf{g}}{3u_1^2}. \quad (12)$$

DM (12) of an atom arises due to a difference of the mean TDMs induced by lower and upper atoms. But this estimate does not take the randomness of positions of atoms in helium into account. The shape [19] of the binary distribution function $g_c(r)$ for He-II implies that the distance between atoms of He-II varies mainly in the interval $R \simeq (5/6 \div 7/6) \bar{R}$, and the mean deviation $\delta R = R - \bar{R}$ satisfies the relation $(\delta R)^2 \simeq (\bar{R}/6)^2$. By introducing the quantity δR in (12), we note that its value is different for different pairs of atoms:

$$R_1 = \bar{R} + \delta R_1 - \frac{\bar{R}}{2} \frac{\partial \bar{R}}{\partial z}, \quad (13)$$

$$R_2 = \bar{R} + \delta R_2 + \frac{\bar{R}}{2} \frac{\partial \bar{R}}{\partial z}. \quad (14)$$

Then

$$\mathbf{d}_g \equiv \left\langle \mathbf{i}_z d_0 \left(\frac{\bar{R}_0^7}{R_1^7} - \frac{\bar{R}_0^7}{R_2^7} \right) \right\rangle = \mathbf{i}_z d_0 \left(\langle f_1^{-7} - f_2^{-7} \rangle + \langle f_1^{-8} + f_2^{-8} \rangle \frac{7}{2} \frac{\partial \bar{R}}{\partial z} \right), \quad (15)$$

$$f_j = 1 + \delta R_j / \bar{R}. \quad (16)$$

With regard for $\langle f_1^{-J} \rangle = \langle f_2^{-J} \rangle$ and (7), we obtain

$$\mathbf{d}_g \approx -\frac{7d_0\bar{R}\mathbf{g}}{3u_1^2} \langle f_1^{-8} \rangle, \quad (17)$$

where

$$\langle f_1^{-8} \rangle \approx 1 + \frac{9!}{7!2!} \left\langle \left(\frac{\delta R}{\bar{R}} \right)^2 \right\rangle + \frac{11!}{7!4!} \left\langle \left(\frac{\delta R}{\bar{R}} \right)^4 \right\rangle + \frac{13!}{7!6!} \left\langle \left(\frac{\delta R}{\bar{R}} \right)^6 \right\rangle + \dots \approx 2.3. \quad (18)$$

Above, we used $\langle (\delta R)^{2J+1} \rangle = 0$ and assumed $\langle (\delta R)^{2J} \rangle = \langle (\delta R)^2 \rangle^J$. The exact formula for the DM, which arose in an atom with the coordinate $z = 0$ due to the presence of $\nabla_z n$ or $\nabla_z T$ in the medium, is as follows:

$$\mathbf{d} = -\mathbf{i}_z d_0 \int_{z>0} n(\mathbf{r}) g_c(\mathbf{r}) \cos \theta \frac{\bar{R}_0^7}{r^7} d\mathbf{r} + \mathbf{i}_z d_0 \int_{z<0} n(\mathbf{r}) g_c(\mathbf{r}) \cos \theta \frac{\bar{R}_0^7}{r^7} d\mathbf{r}. \quad (19)$$

It is necessary to bear in mind that n and T can vary in the corresponding half-space ($z > 0$ or $z < 0$). Therefore, g_c depends on r and z . Since g_c has maximum at $r \approx \bar{R}$, and the main contribution ($\sim 90\%$) to integrals (19) is given by g_c at $r \lesssim 1.5\bar{R}$, we estimate \mathbf{d} , by assuming that values of n and T in the whole upper half-space are equal to those at $z = \bar{R}$ ($z = -\bar{R}$ for the lower half-space). In this case, $g_c(r, z) \equiv g_c(r)$, where $g_c(r)$ corresponds to the indicated n and T which are different in the upper and lower half-spaces. Then

$$\mathbf{d} = -\frac{\mathbf{i}_z d_0}{4n_0} (n S_7) \Big|_{z=-\bar{R}}^{z=\bar{R}}, \quad (20)$$

$$S_j = \int_{\Omega=4\pi} n_0 g_c(r) \frac{\bar{R}_0^j}{r^j} d\mathbf{r}, \quad n_0 = \bar{R}_0^{-3}. \quad (21)$$

Since $S_7 = S_7(T, n)$, we obtain

$$\mathbf{d} = \mathbf{d}_T + \mathbf{d}_\rho, \quad (22)$$

$$\mathbf{d}_\rho = -\frac{S_7 d_0 \bar{R} \nabla_z n}{2n} \left(1 + \frac{n}{S_7} \frac{\partial S_7}{\partial n} \right), \quad (23)$$

$$\mathbf{d}_T = -\frac{d_0 \bar{R}}{2} \frac{\partial S_7}{\partial T} \nabla_z T. \quad (24)$$

The dependence of the structural factor $S(k)$ on the temperature is determined by the formula [18]

$$S(k, T) = S(k, T_0) \coth(E/2T) / \coth(E/2T_0). \quad (25)$$

In order to calculate $S_7(T)$, it is necessary to know the function $g_c(r, T)$. It can be found from $S(k, T)$ with the help of the well-known equation

$$S(k) = 1 + n \int (g_c(r) - 1) e^{-i\mathbf{k}\mathbf{r}} d\mathbf{r}. \quad (26)$$

The numerical calculation of S_7 by (21) was carried out at temperatures from 0 to T_λ , and the experimental function $S(k, T_0 = 1\text{ K})$ from [19] was chosen as a “bare” one, $S(k, T_0)$. It turned out that S_7 depends very weakly on the temperature, so that $\partial S_7 / \partial T = 6 \cdot 10^{-4} K^{-1}$ with a high accuracy for $S_7 \approx 14.879$ in the interval $T = 1\text{ K} \div T_\lambda$. This determined the value of \mathbf{d}_T (24).

Consider \mathbf{d}_ρ (23), by setting $\nabla T = 0$. As a consequence of the relation $g_c(n_2, r) = g_c(n_1, (n_2/n_1)^{1/3}r)$ we obtain

$$S_j(n_2) = \int_{4\pi} n_0 g_c \left(n_1, \left(\frac{n_2}{n_1} \right)^{1/3} r \right) \frac{\bar{R}_0^j}{r^j} d\mathbf{r} = \left(\frac{n_2}{n_1} \right)^{j/3-1} S_j(n_1), \quad (27)$$

$$\frac{\partial S_7}{\partial n} = \frac{4S_7}{3n}. \quad (28)$$

Taking into account (23), we find

$$\mathbf{d}_\rho \approx -\frac{7S_7 d_0 \bar{R} \nabla_z n}{6n}. \quad (29)$$

Therefore, we obtain with regard for (6) that the DM of a helium atom induced by the gravity force is

$$\mathbf{d}_g \approx -\frac{7S_7 d_0 \bar{R} \mathbf{g}}{6u_1^2}, \quad (30)$$

which agrees with (17) and (18). Respectively, the polarization of He-II due to the gravitational field is

$$\mathbf{P}_g = n\mathbf{d}_g \simeq -\frac{7S_7 d_0 \nabla \rho}{6R^2 \rho} = \gamma_\rho \mathbf{g}, \quad \gamma_\rho \approx -\frac{7S_7 d_0}{6R^2 u_1^2}. \quad (31)$$

The relation $\mathbf{d}_\rho = K \nabla \rho / \rho$ can be obtained also [20] on the basis of formulas of work [12]. In this case, the coefficient K corresponds approximately to (29). The formula close to (29) was obtained in [15].

A different result was found in work [6] on the basis of classical “inertial” mechanism:

$$\mathbf{P}_g \equiv n\mathbf{d}_{g,i} = \gamma_i \mathbf{g}, \quad \gamma_i \approx \frac{\varepsilon - 1}{4\pi} \frac{m}{2Z|e|}, \quad (32)$$

where $Z|e|$ is the charge of the atom nucleus ($-2e$ for He^4), ε is the dielectric permittivity (1.057 for He-II), m is the mass of an atom, the index i means “inertial”. According to [6], the gravitational field is a source of the polarization. The relation (32) is similar to the founded above formula (31). But, at $u_1 = 240\text{ m/s}$, the value of γ_ρ is 136 times as large as the value of γ_i (32) and has the different sign. Moreover, we should like to emphasize a more important point: according to the assumption made in [6], polarization (32) is induced directly by the gravitational field, whereas the polarization (31) is induced by the concentration gradient.

These are the different sources of the effect. We will compare both of the approaches in more detail below in Sec. 4.

It is worth noting that analogous equations connecting \mathbf{P} with $\nabla\rho$ and ∇T , follow from the equations for a nonsuperfluid liquid. Hence, the specificity of superfluidity is not manifested there.

Formula (31) determines the polarization of He-II by the gravitational field (the gravielectric effect). It turns out that this polarization is measurable. Let He-II be positioned in a cylindrical dielectric vessel characterized by the vertical axis, height L , and radius R . Let the vessel be completely filled with helium. Let the Z be directed upward. The potential difference $\Delta\varphi$ between point A at the top of the vessel at the center of the surface of helium and point B on the bottom at the center of the base of the vessel can be found analogously to [8] (by considering that the dipoles are positioned only in the volume of helium and by neglecting the polarization of the vessel):

$$\begin{aligned}\Delta\varphi &= \varphi_A - \varphi_B = -2\varphi_B = 2 \int_0^L dz \int_0^R \varrho d\varrho \int_0^{2\pi} d\phi \frac{P_z(z)z}{\varepsilon_{He}(\varrho^2 + z^2)^{3/2}} = \\ &= \frac{4\pi P_z}{\varepsilon_{He}}(L + R - \sqrt{L^2 + R^2}), \quad P_z = -\gamma_\rho g.\end{aligned}\tag{33}$$

In experiments, the sizes of a vessel were approximately as follows: $L = 20$ cm, $R = L/2$. With these values and (33), we get $|\Delta\varphi| \approx 57$ nV. The real $|\Delta\varphi|$ will be somewhat less due to the polarization of walls. It is a very small voltage, but we indicate that a less voltage, $\Delta\varphi = 10$ nV, was measured in works [1, 2]. However, it was alternating.

In the analysis, we used implicitly the assumption that, on the microlevel, the atoms “feel” the density gradient. In the formulas obtained above, sometimes \bar{R} was the average over the time, $\langle R(z) \rangle_t$ and sometimes — over atoms, $(V/N)^{1/3}$. The atoms in helium are moving chaotically, and the question arises whether these averages are equal to each other, in particular, at $\partial\rho/\partial z \neq 0$. That is, is the relation

$$\langle R(z) \rangle_t \equiv \bar{R}(z) = (V_z/N_z)^{1/3}\tag{34}$$

true? Here, V_z is the volume of a thin layer with the given z , and N_z is the number of atoms in the layer. If (34) is satisfied, then the atoms “feel”, on the microlevel, the density gradient, and formulas (13), (20) and those following from them are valid. If (34) would not be satisfied, then $\langle R(z) \rangle_t$ would be different for different atoms even along a layer $z = \text{const}$, where the density is constant. However, the total N -particle wave function of helium-II is symmetric relative to a permutation of atoms both in the ground state and in the presence of quasiparticles [21]. For $\partial\rho/\partial z \neq 0$, the symmetry is conserved in the plane $z = \text{const}$. Therefore, $\langle R(z) \rangle_t$ for different atoms of a layer $z = \text{const}$ must be equal to one another and, hence, to the instantaneous value of the average over atoms $(V_z/N_z)^{1/3}$. That is, (34) is valid.

3 POLARIZATION OF He-II BY A SECOND-SOUND WAVE

A standing second-sound half-wave

$$T = T_0 - 0.5\Delta T \cos(\omega_2 t) \cos(z\pi/L) \quad (35)$$

was realized in experiments in [1]. Here, $L = \lambda_2/2$ is the length of a resonator, and ω_2 is the second-sound frequency. A second-sound wave induces the variable gradient of T in helium by (35), which leads to the appearance of the variable gradient of density. The latter can be determined from the coefficient of thermal expansion α_T by the formula [16, 17]

$$\nabla \rho = -\rho \alpha_T \nabla T (1 - u_2^2/u_1^2)^{-1}. \quad (36)$$

Relations (22)–(24) and (36) together with $u_2 \ll u_1$ allow one to determine both the mean DM arising in every atom of He-II due to the gradients of ρ and T ,

$$\mathbf{d} = \mathbf{d}_\rho + \mathbf{d}_T \approx 0.5d_0 \bar{R} \nabla T (7S_7\alpha_T/3 - \partial S_7/\partial T), \quad (37)$$

and the polarization of He-II,

$$\mathbf{P} = \mathbf{P}_\rho + \mathbf{P}_T \equiv n\mathbf{d}_\rho + n\mathbf{d}_T \approx 0.5d_0 \bar{R}^{-2} \nabla T (7S_7\alpha_T/3 - \partial S_7/\partial T). \quad (38)$$

At $T = 1.3 \div 2 K$, we have $\partial S_7/\partial T = 0.6 \cdot 10^{-3} K^{-1}$, which is much less than $-7S_7\alpha_T/3 = (21.9 \div 415)10^{-3} K^{-1}$ (by the data on α_T [16]). Therefore, the quantity $\partial S_7/\partial T$ in (38) can be omitted. Thus, the polarization in a second-sound wave due to ∇T is much less than the polarization due to $\nabla \rho$.

For the experiment performed in [1], we obtain that the potential difference U between the electrode and the ground is analogously [8]:

$$U(T) = \eta^*(T) \gamma_b(R/L) \cos(\omega_2 t), \quad (39)$$

$$\Delta\varphi(T) \equiv 2\eta^*(T) \gamma_b(R/L) = |-7S_7\alpha_T/3 + \partial S_7/\partial T| \frac{\pi \gamma_b(R/L) d_0}{2\varepsilon_{He} \bar{R}^2} \Delta T, \quad (40)$$

where $\Delta\varphi$ is the amplitude of oscillations of U . At $T = 1.3 K$, we have $\frac{\Delta\varphi}{\Delta T} \approx \frac{\gamma_b(R/L) k_B}{60 \cdot 2e}$, whereas the experiment gives $\frac{\Delta\varphi}{\Delta T} \approx \frac{k_B}{2e}$. Here, the factor $\gamma_b(R/L) \equiv \gamma_{bound}$ is not related to the polarization (i.e., not to the relation $\mathbf{P} = -\gamma\mathbf{w}$), but it is determined by boundary conditions (see [8]). For a short resonator [1], $\gamma_b \simeq 1.38$; therefore $\frac{\Delta\varphi}{\Delta T} \approx \frac{1}{45} \frac{k_B}{2e}$, which takes 1/45 of the experimental value. In the case of a long resonator, $\gamma_b \simeq 0.05$, and $\frac{\Delta\varphi}{\Delta T} \approx \frac{1}{1200} \frac{k_B}{2e}$. At $T = 1.8 K$, we have $\frac{\Delta\varphi}{\Delta T} \approx \frac{1}{4.5} \frac{k_B}{2e}$ and $\frac{\Delta\varphi}{\Delta T} \approx \frac{1}{120} \frac{k_B}{2e}$ for the short and long resonators, respectively.

As T increases, the theoretical quantity $\Delta\varphi/\Delta T$ grows like α_T , whereas the experimental one $\Delta\varphi/\Delta T$ does not depend on T and the ratio R/L of the sizes of a resonator. Thus, the above-presented bulk mechanism predicts a weak electric signal which is by 1-3 orders lower than the observed signal and strongly depends on the sizes of a resonator and the temperature. This signal can be measured under a higher accuracy of measurements.

We note two points related to the electric signal arising due to the volume polarization of He-II. First, such a signal must obligatorily depend on the sizes of a resonator. Moreover, this dependence would be quite strong due to the factor γ_b . Indeed, the volume polarization of helium is a result of the polarization of separate atoms, since helium contains nothing except for atoms. The atoms possess a tidal DM. Therefore, since the polarization is determined by the sum of atomic DMs, the factor γ_b is sure to appear, and the signal $\Delta\varphi$ would turn out to be strongly dependent on the length of a resonator, which was not observed in experiments. If the temperature of the whole resonator would be noticeably changes synchronously with second sound, this would induce oscillations of T in helium outside of the resonator and the corresponding polarization. In this case, $\Delta\varphi$ would be formed in the volume of $\sim L^3$, and, hence, γ_b and $\Delta\varphi$ would be close to γ_b and $\Delta\varphi$ for a short resonator. That is, $\Delta\varphi$ would be weakly dependent on the sizes of a resonator. But the experiment [22] indicates that the oscillations of T in a second-sound wave do not enter practically into helium outside of a resonator due to thick walls and a low heat conduction of a resonator.

The second point consists in that the term $\sim \partial S_7 / \partial T$ in formulas (38) and (40) describes the polarization due to ∇T (at $\nabla\rho = 0$). Since S_7 is expressed through the structural factor, this term involves all possible microscopic mechanisms leading to the volume polarization due to ∇T . But the contribution of this term to the polarization is small. Therefore, such mechanisms cannot explain the observed signal $\Delta\varphi$.

Thus, the electric signal $\Delta\varphi$ induced by a second-sound wave must consist of two parts: the major one, $\Delta\varphi_S$, and the minor one, $\Delta\varphi_V$. The signal $\Delta\varphi_S$ observed in the experiment [1] was apparently caused by some surface effect in helium or by the gradient thermoemf arising in the electrode due to a gradient of temperatures. The quantity $\Delta\varphi_S$ is independent of the sizes of a resonator and T and is a response only to variations of the temperature. But the quantity $\Delta\varphi_V$ is the above-discussed signal which has not been yet discovered experimentally. It appears due to the volume polarization of He-II and depends strongly on the sizes of a resonator. The shorter a resonator, the stronger this effect. Therefore, it is easier to discover the effect with short resonators (where $L \lesssim R$), and the resonator must be fabricated of a dielectric (in order to avoid induced noises and to increase γ_b).

4 CONNECTION BETWEEN ACCELERATION AND POLARIZATION

One of the goals of the present work is the study of the connection between the polarization and the acceleration for the atom or medium. This problem was considered in the work [6], in which it was found that the polarization \mathbf{P} of a dielectric is proportional to its acceleration \mathbf{w} :

$$\mathbf{P}_i = -\gamma_i \mathbf{w} \quad (41)$$

with γ_i from (32). According to [6], the acceleration \mathbf{w} is a source of the polarization. However, the reasoning underlying formula (41) is not quite correct, in our opinion. It was assumed in the derivation of (41) that the electron shell of a single atom of the quiescent dielectric is stretched by the gravitational field. But the gravity leads to the identical acceleration for electrons and the nucleus. Therefore, it cannot stretch any atom. Hence, the reasoning in [6] and formula (41) lose the significance. In addition, in [6], the interaction of a single atom with the other atoms was considered only in terms of ε , i.e. on the macroscopic level; but such a situation should be analyzed microscopically, by explicitly taking the interaction of the atom with its neighbors into account.

Let us assume that the gravity does stretch an atom. We will show that, in this case, the explicit consideration of its interaction with the neighboring atoms is of importance. According to [6], a quiescent dielectric in a gravitational field acquires the polarization (32), which yields (41). The brief substantiation of formula (32) can be found in [6] and, in more details, in [14]. The reasoning of works [6, 14] is as follows. The nucleus of an atom of the dielectric undergoes the action of the gravity force $m_c \mathbf{g}$ and somewhat “sags” by the distance δ_g . As a result, the atom acquires a DM of $Z|e|\delta_g$, and the dielectric becomes polarized. For small deformations $\delta_g = \eta m_c \mathbf{g}$, η is determined from the following relations for an atom in an electric field: $d = \kappa E/n$, $d = Ze\delta_E = Ze\eta(2ZeE)$, where $\kappa = (\varepsilon - 1)/4\pi$. From this relations, we get $\eta = \frac{\kappa}{2n(Ze)^2}$ and formula (32). In such an approach, the gravitational field acting on the atom nucleus, is compensated by the force \mathbf{F}_d related to the elastic deformation of the electron shell of the atom:

$$\mathbf{F}_d + m_c \mathbf{g} = 0, \quad \mathbf{F}_d = -\delta_g / \eta. \quad (42)$$

Let us consider the problem in more details. We can easily evaluate the ratio of the forces affecting an atom in a dielectric, by considering, firstly, a dielectric without the gravity and then by “switching-on” the gravity force. Without the gravity, only the interatomic forces act on the atoms of a dielectric. From the side of each of the neighboring atoms, a specific atom is affected by: 1) the van der Waals force \mathbf{F}_v which is well-known at $R \gtrsim a$ for helium-II [23]:

$$\mathbf{F}_v = -\nabla U, \quad U(R) = 4\varepsilon \left(\left(\frac{a}{R} \right)^{12} - \left(\frac{a}{R} \right)^6 \right), \quad \varepsilon = 11K, \quad a = 2.64\text{\AA}, \quad (43)$$

where R is the interatomic distance, a is the “size” of an atom; 2) the force of “quantum pressure” (arising due to the motion of atoms caused by zero-point oscillations which increase the mean energy of an atom at $T = 0$ from $-\varepsilon = -11K$ to $E_0 = -7.16K$) equal approximately to $F_{qp} \simeq \Delta p / (\Delta t) \simeq 2\langle p \rangle / (\Delta t) \simeq \langle p \rangle^2 / (m_4(R - a)) \simeq \hbar^2 / (m_4(R - a)^3)$; 3) the pressure due to the thermal motion of atoms.

Let us switch-on the gravity force. It gives the same acceleration to electrons and the nucleus. So, each atom of a dielectric starts to move downward, and the dielectric will be contracted. He-II has a high heat conductivity; therefore, the temperature and the thermal pressure at all points of a vessel will be fast equalized. In this case, the gravity force must be

compensated owing to that the interatomic force $\mathbf{F}_v + \mathbf{F}_{qp}$ acting on every atom from the side of lower neighboring atoms exceeds the analogous force from the side of upper neighboring atoms by m_4g . To realize this, the gradient density should be present in a dielectric. How can a DM of helium atoms be determined in this case? In [6, 14], the DM was determined on the basis of the assumption that the atom is elongated in the gravitational field by δ_g so that the gravity force acting on the nucleus is compensated by the elasticity of an atom according to (42). We now study whether the atom will be elongated in such a way. To this end, we compare the elastic force with interatomic forces. Let the atom move downward by a distance of δ_g . This results in the appearance of the difference of the distances to the neighboring upper and lower atoms $\delta\bar{R} = 2\delta_g$, as well as the difference of the van der Waals forces, $\delta F_v = F_v(\bar{R}) - F_v(\bar{R} - 2\delta_g) = F'_v(\bar{R})2\delta_g = -U''(\bar{R})2\delta_g$. At the tension by δ_g , the elastic force is $F_d = m_cg \approx m_4g$ according to (42). In view of (43), we obtain

$$\begin{aligned} \frac{\delta F_v}{F_d} &= -\frac{U''(\bar{R})2\delta_g}{m_4g} = -2\eta U''(\bar{R}) = \\ &= -\frac{8\varepsilon\eta}{\bar{R}^2} \left(\left(\frac{a}{\bar{R}} \right)^{12} 12 \cdot 13 - \left(\frac{a}{\bar{R}} \right)^6 6 \cdot 7 \right) \approx 2.9 \cdot 10^{-6}, \end{aligned} \quad (44)$$

where $\bar{R} = \bar{R}_0 = 3.578 \text{ \AA}$ is the mean distance between atoms of He-II at $\rho = 0.1452 \text{ g/cm}^3$. That is, the difference of the van der Waals forces arisen due to the lowering of an atom at a distance of δ_g is *by 6 orders less* than the forces owing to the atomic elasticity appearing at the tension of an atom by the same δ_g . This implies that the atom is a very elastic object and will fall down without tension. In the above estimates, we neglect the quantum pressure, whose contribution is of the same order as that of the van der Waals force, but its correct modeling is a difficult task.

Thus, the atom in a gravitational field must not be stretched by δ_g , because the gravity force affecting the nucleus is compensated by the difference of interatomic forces for the adjacent layers of atoms, rather than by the tension of an atom. A slight tension is possible, but it will lead to a polarization significantly less than (32), because we have $\delta_g/\eta \ll m_cg$ instead of $\delta_g/\eta = m_cg$. Therefore, γ_i in formulas (41) and (32) describing the inertial polarization related to the elastic tension of an atom must be significantly less than γ_i given in (41). While studying the accelerated motion of a dielectric, we need also to consider the interaction of atoms, because the inertia forces do not exist in the nature and are used only as a convenient mean to describe the accelerated motion of bodies caused by the real forces (for the atoms of He-II, they are interatomic forces).

The estimates indicate that the inertial mechanism of the polarization must take the interaction of atoms into account and, hence, must be developed in the frame of quantum theory. Let us consider a single atom. To move with acceleration, it should undergo the action of an external force, e.g., the gravity force or the electromagnetic wave. In both cases, the atom will not be polarized by acceleration, because the inertial forces (and the gravity force) accelerate electrons and the atomic nucleus identically. But the source of a force can be

another atom. This is a partial case with a nonzero density gradient where two immovable He atoms polarize each other according to (9). Let the distance between the atoms be somewhat larger than the equilibrium one (corresponding to the minimum of the potential (43)). In this case, the equilibrium does not hold, and the atoms begin to approach each other. As a result, the tidal DM of each atom is changed. By expanding the DM in a series in the small displacement δR , velocity, acceleration w , etc., we can evaluate the contribution of these parameters to the DM of an atom with the help of nonstationary perturbation theory. In this case, the acceleration is the source of the corresponding correction to the DM, and it is natural to name this correction the inertial polarization of an atom. The preliminary analysis showed that it $\sim w^2$. That is, this correction is small, and the zero approximation (9) (for immovable atoms) gives the main contribution. In the derivation of formulas (23) and (24), the atoms were considered immovable. We did not calculate the inertial correction to a macroscopic polarization and *consider the question about the value of γ_i in (41) to be open*. Most probably, $|\gamma_i| \ll |\gamma_\rho|$, but we do not exclude completely the possibility that $|\gamma_i| \gtrsim |\gamma_\rho|$.

In the zero approximation (i.e. for immovable atoms), the connection between the polarization \mathbf{P} and the acceleration \mathbf{w} of the medium can be determined from formulas (31) and (24). According these formulas, if $\nabla\rho$ and/or ∇T are nonzero in the medium, the medium is polarized. It is obvious from (2) and (5) that if the gravity force is absent, but there is the constant gradient of density in He-II (6), then helium as a whole must move with the acceleration $\mathbf{w} = -\mathbf{g}$. In the reference system where helium is at rest, relation (31) holds. It allows us to get the connection between the polarization of helium and its acceleration:

$$\mathbf{P}_\rho = -\gamma_\rho \mathbf{w}_\rho, \quad \gamma_\rho \simeq -\frac{7S_7 d_0}{6R^2 u_1^2}. \quad (45)$$

We now determine the connection of the polarization \mathbf{P}_T appeared due to the gradient of T with the corresponding acceleration \mathbf{w}_T . It is seen from the equations

$$\rho \mathbf{w} \equiv \rho D\mathbf{v}/Dt = -\nabla p, \quad (46)$$

$$\nabla p = \frac{\partial p}{\partial \rho}|_T \nabla \rho + \frac{\partial p}{\partial T}|_\rho \nabla T \approx u_1^2 \nabla \rho + \frac{\partial(SV)}{\partial V}|_T \nabla T, \quad (47)$$

$$\begin{aligned} \frac{\partial(SV)}{\partial V}|_T &= -\frac{\rho}{V} \frac{\partial(SV)}{\partial \rho}|_T = -\frac{\rho}{V} \frac{\partial(SV)}{\partial p}|_T \frac{\partial p}{\partial \rho}|_T \approx \\ &\approx \frac{\rho u_1^2}{V} \frac{\partial^2(FV)}{\partial p \partial T} = \frac{\rho u_1^2}{V} \frac{\partial V}{\partial T}|_p = \rho u_1^2 \alpha_T \end{aligned} \quad (48)$$

that the acceleration of a fluid element is given by two terms which are related, respectively, to $\nabla\rho$ and ∇T :

$$\mathbf{w} \approx -u_1^2 \nabla \rho / \rho - u_1^2 \alpha_T \nabla T \equiv \mathbf{w}_\rho + \mathbf{w}_T. \quad (49)$$

Relations (49) and (23) yield (45), and relations (49) and (24) allow us to get

$$\mathbf{P}_T = -\gamma_T \mathbf{w}_T, \quad \gamma_T \approx \frac{\partial S_7}{\partial T} \frac{d_0}{2u_1^2 \alpha_T R^2}. \quad (50)$$

In this case, $\gamma_\rho \simeq -136\gamma_i$ and $\gamma_T(T = 1.3 \div 2 K) \simeq (0.017 \div 0.001)\gamma_\rho$.

The acceleration in a second-sound wave, according to relations (36) and (49), is equal to $\mathbf{w}_2 \approx -u_2^2 \nabla \rho / \rho$ [16, 17]. That is, the accelerations induced by the gradients of ρ and T are almost the same by modulus and of the opposite signs. In this case, the polarizations \mathbf{P}_ρ and \mathbf{P}_T do not compensate each other, since $P_T \ll P_\rho$. For a second-sound wave, the total polarization and acceleration are connected by the relation

$$\mathbf{P} = \mathbf{P}_\rho + \mathbf{P}_T = -\gamma_2 \mathbf{w}_2, \quad \gamma_2 \approx (u_1/u_2)^2 \gamma_\rho, \quad (51)$$

so that $\gamma_2 \sim 10^2 \gamma_\rho \sim -10^4 \gamma_i$.

It is seen from formulas (45), (50), and (51) that there is no universal connection between \mathbf{P} and \mathbf{w} in the medium, and the value of γ depends on the mechanism of the polarization. In addition, despite the fact that the polarization is proportional to the acceleration, the gradient $\nabla \rho$ or ∇T is the *source* of a polarization, rather than the acceleration or the gravity field (31). The acceleration is another consequence of $\nabla \rho$ (or ∇T).

The tidal DMs and higher multipoles of atoms arising in all these cases induce the arbitrarily small forces (as compared with the van der Waals force) in the medium. Therefore, the tidal polarization of atoms should not be included in the equations of two-fluid hydrodynamics; it does not affect $\nabla \rho$ or ∇T and cannot compensate the gravity field. The tidal DM is similar to a vane which shows the direction of a wind (the direction to the nearest atom) but does not affect it.

Thus, though the polarization of individual atoms is induced by the tidal mechanism and not by acceleration, however, at nonzero $\nabla \rho$ and/or ∇T the spatial location of the atoms is such that instantaneous distances between them do not correspond to the equilibrium ones, so that interaction between the atoms leads to their acceleration \mathbf{w} , and an average polarization of the atoms turns out to be proportional to the average value of \mathbf{w} , as was first suggested in [6].

Apparently, for any microscopic origin of \mathbf{w} , there is the linear connection $\mathbf{P} = -\gamma \mathbf{w}$ if \mathbf{w} is small. Indeed, both the polarization and the acceleration are caused by the directed inhomogeneity of the position or motion of atoms, the values of \mathbf{P} and \mathbf{w} are proportional to this inhomogeneity and, hence, to each other.

We should like to note that work [6] became one of the stimuli for the present study.

5 CONCLUSIONS

In the present work, the polarization of He-II due to the gradients of density and temperature is calculated, the relation of the polarization to the acceleration of the medium is studied, and the gravielectric effect is considered. It is shown that a second-sound wave should induce a volumetric electric signal $\Delta\varphi$ mainly arising due to the gradient of ρ in helium. This signal is essentially less than the registered one and can be observed within the future more exact

measurements. As for the Rybalko's effect (i.e., the observed signal $\Delta\varphi \simeq k_B\Delta T/2e$), it is caused, apparently, by a surface mechanism.

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